



## **A Finite Element Study of Beam on Reinforced Granular Beds with Sand Drains**

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**ABSTRACT:** This paper presents the settlement analysis of a beam resting on geosynthetic reinforced granular fill-soft soil system. Each subsystem of the reinforced fill-soil system is idealized by elastic membrane, Pasternak shear layer, Winkler springs and dashpots, as applicable. The suggested model incorporates various aspects of the behavior of the geosynthetic-reinforced granular fill-soft soil system such as horizontal stress induced in the granular fill, the compressibility of the granular fill, and the time-dependent behavior of the subgrade. The differential equations governing the settlement response of the beam resting on two layered reinforced foundation soil has been formulated by incorporating deformation compatibility conditions. The numerical solutions are obtained using Finite Element Method and results are presented in non-dimensional form. The parametric studies are carried out to enumerate the effects of parameters on the settlement response of the system. Results indicate that over a large number of various parameters under large deformation the proposed model evaluate the settlement of the system and horizontal displacement of membrane with reasonable accuracy. It is observed that compressibility, shear modulus and thickness of granular fill, pre-stressing and tension modulus of reinforcement have appreciable influence on the settlement of the system and horizontal displacement of membrane. It is observed that the horizontal displacement of the geosynthetic membrane is negligible as compared to that of the vertical settlement. The model is also analyzed for the case of a sand drain in the soft soil which indicated that the elapsed time and the radius of the sand drain significantly affects the settlement response of the system.

### **1 Introduction**

A foundation constructed on soft soils may experience excessive settlement and possible bearing capacity failure under a surcharge load. One technique that is mostly used nowadays to improve the strength of soft foundation soils is the placement of engineered granular fills containing geosynthetic reinforcement (e.g. geogrid, geotextiles) on the soft soil. Model tests carried out by various researchers (Fragaszy and Lawton, 1984; Love *et al.* 1987; Mandal and Sah, 1993; Adams and Collin, 1997) exhibited improved behavior of the soft soil system with respect to their load settlement response whenever a geosynthetic, with or without pretension, is provided along with the granular fill. The calculation of settlement and ultimate bearing capacity of the geosynthetic reinforced granular fill over soft soil, along with the requirement of stability in terms of settlement, is an important issue for the design engineers. Such issues can be, and are commonly taken care of by using the approach of analyzing the beams on elastic foundations and obtaining the flexural response of the system under different loading conditions, and by using different solution techniques.

Several concepts have been developed to explain the reinforcing mechanism of the reinforcement used in the soil. One of the approaches to study the interaction problem is by idealizing the behavior of the soil. The linear elastic idealization of the supporting soil medium is usually represented by one-parameter or two-parameter mechanical or mathematical model, such as Winkler model, Filolenko – Borodich model, Pasternak model, Kerr model (Kerr, 1964) etc. Over times, various combinations of the above simplified models with varying degree of complexity has come into existence in order to study the influence of the material properties of the soil such as shape, size, configuration, stress history, soil moisture, and permeability on the behavior of foundation systems. Several analytical and numerical works have been carried out on this aspect by various researchers (Biot, 1937; Gazis, 1958; Vesic, 1961; Cheung and Nag, 1968; Rao *et al.*, 1971; Sharma and Dasgupta, 1975; Giroud and

Noiray, 1981; Zhaohua and Cook, 1983; Mastuda and Sakiyama, 1987; Razaqpur and Shah, 1991; Gendy and Saleeb, 1999; Yin, 2000).

The present study attempts to give an insight to the settlement analysis of a beam resting on a reinforced granular fill supported on soft clay with sand drain. In this study, the footing is idealized as an elastic beam. A mechanical foundation model for the reinforced granular fill-soft soil system is considered to incorporate the compressibility as well as the time-dependent behavior of the foundation system. The various parts of the geosynthetic reinforced granular fill-soft soil system are idealized by mechanical elements as shear layer, rough membrane, Winkler springs and viscous dashpots. The equations governing the settlement response of the beam resting on the geosynthetic reinforced granular fill soft soil system have been derived considering the equilibrium of shear layer and rough elastic membrane. The numerical solution has been obtained using finite element method. Since most of the problems have been solved using models like Winkler, Pasternak etc., the same approach has been adopted and the FEM approach has been incorporated in this study as this is a simpler approach than finding out equivalent properties of the reinforced layer. The settlement behavior of the system has been studied for various loading intensities acting over the span of the beam with variation of different soil parameters, effect of pre-stressing the geosynthetic reinforcement, rigidity of the beam and degree of consolidation.

## 2 Definition and formulation of the problem

Figure 1(a) depicts a geosynthetic reinforced granular fill on soft foundation soil supporting a beam. The foundation consists of a granular fill overlying the soft foundation soil. A layer of geosynthetic has been provided as reinforcement in the granular fill. An equivalent mechanical model supporting the beam is shown in Figure 1(b) as proposed by Shukla and Chandra (1994). In this model, a stretched, rough, elastic membrane represents the geosynthetic reinforcement and the Pasternak shear layer represents the granular fill. The compressibility of the granular fill is represented by a layer of Winkler springs (spring constant  $k_f$ ) attached to the bottom of the Pasternak shear layer. The membrane divides the shear layer into two parts. The saturated soft foundation soil is idealized by the Terzaghi consolidation model, which has springs and dashpots attached to the bottom of the Pasternak shear layer. The springs represent the soil skeleton and the dashpots simulate the dissipation of excess pore water pressure in soil.

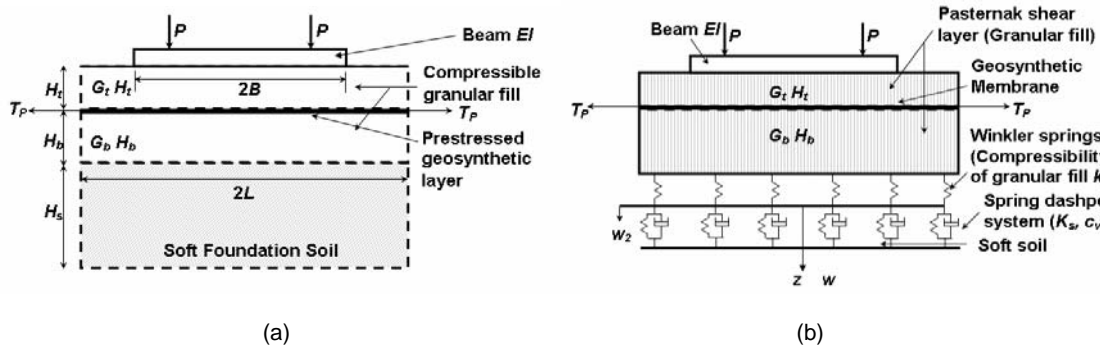


Figure 1. Definition sketch and Schematic diagram of the problem.

The geosynthetic is placed inside the granular fill, for which the granular layer above the reinforcement has a shear modulus value  $G_t$  and the granular below the reinforcement has the shear modulus value  $G_b$ . The geosynthetic reinforcement has been prestressed with force  $T_p$  in addition to the mobilized tensile force,  $T$ , in the reinforcement under loadings. The general assumptions made in the study are:

1. Geosynthetic reinforcement is linearly elastic, rough enough to prevent slippage at the soil interface and has no shear resistance, and thickness of reinforcement is neglected.
2. A rigid perfectly plastic friction model is being adopted to represent the behavior of the soil-geosynthetic interface in shear.
3. The modulus of subgrade reaction of soil has constant value irrespective of depth and time.

4. The consolidation characteristics of the soil both in the loaded region and beyond it are considered to be the same.
5. The rotation of reinforcement is small.

The equations governing the settlement response of the beam on the geosynthetic-reinforced granular fill-soft soil system at any particular time  $t > 0$  as obtained by Yin (1997) using the following nondimensional parameters:

$$X = \frac{x}{B}, W = \frac{w}{B}, V = \frac{v}{B}, H_t^* = \frac{H_t}{B}, H_b^* = \frac{H_b}{B}, G_t^* = \frac{G_t}{k_s B}, G_b^* = \frac{G_b}{k_s B} \quad (1a)$$

$$E_g^* = \frac{E_g}{k_s B^2}, P^* = \frac{P}{k_s B^3}, uo^* = \frac{uo}{k_s B}, T_p^* = \frac{T_p}{k_s B^2}, T^* = \frac{T}{k_s B^2}, EI^* = \frac{EI}{K_s B^5}$$

are written as:

$$T^* = E_g^* \left[ \frac{dV}{dX} + \frac{1}{2} \left( \frac{dV}{dX} \right)^2 + \frac{1}{2} \left( \frac{dW}{dX} \right)^2 \right] \quad (1b)$$

$$\frac{dT^*}{dX} \frac{dW}{dX} + (T^* + T_p^*) \frac{d^2 W}{dX^2} - \left[ EI^* \frac{d^4 W}{dX^4} + \beta W + \beta u_0^* (1 - UC) \right] + K^1 \frac{d^2 W}{dX^2} = 0 \quad (1c)$$

$$T^* \frac{d^2 W}{dX^2} + T_p^* \frac{d^2 W}{dX^2} - EI^* \frac{d^4 W}{dX^4} - \beta W - \beta u_0^* (1 - UC) + K^1 \frac{d^2 W}{dX^2} + KV \left( \frac{dW}{dX} \right) = 0 \quad (1d)$$

Equations (1a), (1b) and (1c) are the governing differential equations for solving three unknowns  $T, W, V$ . Since the problem to be analyzed is symmetric in terms of loading and geometry, only half of the model needs to be analyzed.

## 2.1 Boundary conditions

The solutions are obtained for normalized point load acting over the beam of width  $2B$ . The slope of the settlement distance profile and mobilized tension distance profile at the centre of the beam is taken as zero. At the end of the reinforced zone the slope of the settlement distance profile is considered as zero, as observed in most of the practical cases, whether the membrane is free or fixed. The mobilized tensile force at the edge of the reinforcement is considered as zero.

## 2.2 Method of solution

Virtual displacements  $\delta W$  in the vertical direction and  $\delta V$  in the horizontal direction are applied. From the principle of virtual work, the total work done is zero. The unknown functions  $V, W$  are obtained approximately using the finite element method. Since the governing equations are non linear in nature, an iterative technique has to be employed (here we use direct iteration or Picard's iteration scheme). An approximation of  $V$  and  $W$  are considered as follows:

$$V = \sum_{j=1}^{ne} N_j V_j, W = \sum_{j=1}^{ne} N_j W_j \quad (2)$$

where,  $\{N\}^e$  is the Lagrangian shape function vector, and  $\{W\}^e$  is elemental vector of vertical displacements.

At  $(i+1)^{th}$  iteration, the virtual displacement equations are linearized by using values of  $\frac{dW}{dX}$  and  $\frac{dV}{dX}$  from the

$i^{th}$  iteration. For the first iteration,  $W_i = V_i = \left( \frac{dW}{dX} \right)_i = \left( \frac{dV}{dX} \right)_i = 0$  is assumed, and the elemental matrices are

obtained assembling elemental matrices, we get

$$[K1]\{W\} = \{F1\}; \sum_{e=1}^{ne} [k1]^e = [K1]; \sum_{e=1}^{ne} [f1]^e = [F1] \quad (3)$$

Equation (3) gives  $W$ .

The global finite element form of the above is expressed as follows and used to obtain the solution for each iteration until the results converge.

$$\left[ \begin{aligned} [K2]\{W\} &= \{F2\}; \sum_{e=1}^{ne} [k2]^e = [K2]; \sum_{e=1}^{ne} \{f2\}^e = \{F2\} \\ [K3]\{V\} + [K4]\{W\} &= \{F3\}; \sum_{e=1}^{ne} [k3]^e = [K3]; \sum_{e=1}^{ne} [k4]^e = [K4]; \sum_{e=1}^{ne} \{f3\}^e = \{F3\} \end{aligned} \right] \quad (4)$$

### 2.3 Convergence Criterion

The solution to the nonlinear problem at an instant of time  $t$  has been obtained with the convergence criterion.

$$\frac{\sqrt{\sum_{j=1}^{nn} (V_{i+1} - V_i)^2}}{\sqrt{\sum_{j=1}^{nn} (V_{i+1})^2}} < \varepsilon; \quad \frac{\sqrt{\sum_{j=1}^{nn} (W_{i+1} - W_i)^2}}{\sqrt{\sum_{j=1}^{nn} (W_{i+1})^2}} < \varepsilon \quad (5)$$

where,  $(i + 1)$  and  $i$  are present and previous iterations respectively;  $j$  is the number of nodes used and  $\varepsilon$  is the specified tolerance, which in present case is taken as 0.0001. The ranges of various normalized parameters studied are shown in Table 1.

Table 1. Ranges of various normalized parameters studied.

Sl. No.	Normalized Parameters	Ranges
1	Load intensity $P^*$	5 – 15
2	Shear modulus $G_t^*, G_b^*$	0.1 – 1.0
3	Spring constant ratio $\alpha$	5.0 - $\infty$
4	Pretension $T_p^*$	0.1 – 1.0
5	Thickness of the shear layer $H_t^*, H_b^*$	0.05 – 5.0
6	Degree of consolidation $UC$	0 – 100 %
7	Tension modulus of geosynthetic membrane $E_g^*$	1 – 200
8	Radius of sand drain $R$	0.1 – 0.5
9	Time of consolidation	10 - 25

### 2.4 Consolidation using sand drains

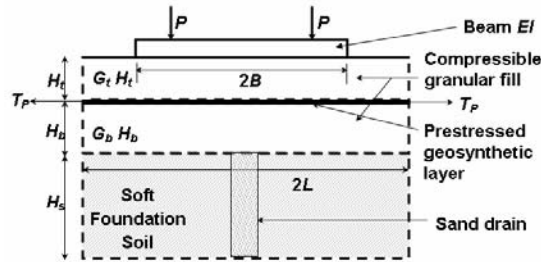


Figure 2. Definition sketch of geosynthetic reinforced granular fill soft soil system with sand drain

The consolidation process of soft subsoil is a process, which continues for very long time due to its low permeability. In order to accelerate the process of consolidation settlement, sand drains are installed in the soft layer. Sand drains are constructed by driving down casing mandrels into the soil. The holes are then filled with sand, after which casings are pulled out. When a surcharge is applied at ground surface, the pore water pressure in the clay increases, thus, initiating hydraulic gradient in the vertical and horizontal directions. The horizontal drainage is induced by sand drains. Hence, the process of dissipation of excess pore water pressure created by the loading is accelerated. The drainage due to vertical sand drain takes place both horizontally and vertically.

The horizontal permeability of clay is generally higher than the vertical permeability. The sand drain is assumed to be incompressible and the ratio of horizontal to vertical permeability of subsoil layer is taken as 2.5.

In the case of sand drains, as the process of 3-D consolidation is symmetrical about the vertical axis, the problem can be simplified as a 2-D problem. In case of equal strain, the differential equation governing the consolidation process is expressed as:

$$\frac{\partial u}{\partial t} = C_{vr} \left[ \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right] + C_{vz} \frac{\partial^2 u}{\partial z^2} \quad (6)$$

Using the following normalized parameters

$$Z = \frac{z}{H}, R = \frac{r}{B}, T_c = \frac{\sqrt{C_{vr} C_{vz}}}{H.B} t, U = \frac{u}{q} \quad (7)$$

where,  $H$  is the depth of soft foundation soil (length of sand drain),  $B$  is the half-width of beam,  $T_c$  is the normalized time,  $C_{vr}$  and  $C_{vz}$  are coefficient of consolidation in radial and vertical direction respectively. Using the normalized parameters given above, equation (6) is written as,

$$\frac{\partial U}{\partial T_c} = \frac{H}{B} \sqrt{\frac{C_{vr}}{C_{vz}}} \left[ \frac{\partial^2 U}{\partial R^2} + \frac{1}{R} \frac{\partial U}{\partial R} \right] + \frac{B}{H} \sqrt{\frac{C_{vz}}{C_{vr}}} \frac{\partial^2 U}{\partial Z^2} \quad (8)$$

The governing differential equation for this problem is expressed as

$$\int_A \frac{\partial U}{\partial T_c} v R dR dZ + \left[ \int_A \alpha^* \frac{\partial U}{\partial R} \frac{\partial v}{\partial R} R dR dZ + \int_A \frac{1}{\alpha^*} \frac{\partial U}{\partial Z} \frac{\partial v}{\partial Z} R dR dZ \right] = 0; \quad \alpha^* = \frac{H}{B} \sqrt{\frac{C_{vr}}{C_{vz}}} \quad (9)$$

The elemental finite element formulation is derived and the global form is expressed as:

$$[A] \{U\}_{i+1} = \{C\}; [A] = [M] + \theta \Delta t_{i+1} [K]; \{C\} = [[M] - (1-\theta) \Delta t_{i+1} [K]] \{U\}_i; \quad (10)$$

$$\sum_{e=1}^{ne} [m]^e = [M]; \sum_{e=1}^{ne} [k]^e = [K]; \dot{U}_{i+1} = \left[ \frac{U_{i+1} - U_i}{\Delta t_{i+1}} - (1-\theta) U_i \right] \frac{1}{\theta}$$

At time  $t = 0$  we have  $U = 1.0$ . Equation (10) is the basic equation for solving unknowns  $\{U\}_{i+1}$  from the known values of  $\{U\}_i$ . The Crank-Nicholson scheme is used for finite difference for which  $\theta = 0.5$ , the above scheme is unconditionally stable. The degree of consolidation obtained is a function of radial distance from center of loading, depth and time, for any particular time the average degree of consolidation at any radial distance is obtained by averaging the degree of consolidation values at that radial distance along the depth. This average degree of consolidation is used as an input for the geosynthetic reinforced granular fill soft soil system to study its response when sand drains are installed in the soft soil.

### 3 Results and discussions

#### 3.1 Effect of tension modulus of geosynthetic reinforcement

Figure 3(a) shows the effect of variation of tension modulus of geosynthetic membrane on the settlement response of the geosynthetic reinforced granular fill soft soil system considering other parameters as constant. It is observed that the variation in the tension modulus of geosynthetic membrane does not appreciably affect the settlement response of the system under the range of parameters studied. Figure 3(b) shows the horizontal displacement profiles for various values of tension modulus of geosynthetic reinforcement. It is observed that as the tension modulus increases, the horizontal displacement decreases.

#### 3.2 Effect of pretension in geosynthetic membrane

Figure 4 shows the effect of pretension in the geosynthetic reinforcement on the settlement behavior of the system. As pretension in the geosynthetic reinforcement is increased, settlement decreases below the point of application of load and no appreciable change takes place at the center of system. As the pretension force increases, heaving near the edge decreases. This results in reduction in differential settlement. It is concluded that pre-tensioning of geosynthetic reinforcement may prove to be very effective in improving the ground when very small differential settlement is desired.

### 3.3 Effect of thickness of granular fill

Figure 5 shows the effect of variation of thickness of granular fill on the settlement response of the system at 90% degree of consolidation and without any pretension. It is observed that settlement decreases with increase in the thickness of granular fill below the point of application of load. There is no appreciable variation in the settlement at the center of the system while settlement increases at the edge of the beam. It can be seen that for higher values of thickness, there is very less variation in settlement. It can be stated that increase in the thickness of granular fill is quite effective to reduce the settlement

### 3.4 Effect of shear modulus of granular fill

Figure 6 depicts the settlement profiles of the system for various values of shear modulus of granular fill. The shear modulus of top and bottom granular layer is kept identical. As the magnitude of the shear modulus of fill increases, settlement decreases below the point of application of load; while it increases at centre of the reinforcement. Variation in settlement at edge of reinforcement is very small. It is also observed that for higher values of shear modulus, variation in settlement along the length of the system is very small. From this fact, it may be concluded that higher values of shear modulus are preferable to avoid differential settlement. It is observed that increase in the magnitude of shear modulus of fill beyond 0.4 has no effect on the settlement characteristics of the system.

### 3.5 Effect of spring constant ratio of granular fill

Figure 7 shows the effect of spring constant ratio on settlement profiles of the system at 90% consolidation with no pretension in the geosynthetic reinforcement, thus bringing out the effect of the relative compressibility of the granular fill and the soft soil on the settlement behavior. The settlement profile plotted for infinite spring constant ratio corresponds to the granular fill being taken as incompressible. It is observed that the settlement at any location decreases as the spring constant ratio is increased. The variation in settlement for the values of spring constant more than 10 is very less along the length of system. As the spring constant ratio increases from 5 to 10 and 20, the decrease in settlement is 9.4% and 15.63% respectively. The decrease in settlement is 1.8% when spring constant increases from 50 to infinity. Therefore it may be concluded that when the granular fill is 50 times stiffer than soft soil, the compressibility of the granular fill can be ignored.

### 3.6 Effect of sand drain

Figure 8(a) shows the settlement profiles at various stages of consolidation. It is observed that as consolidation takes place, the settlement increases at all locations of the system. In the system without sand drains, degree of consolidation was considered uniform throughout the length of the system; but in the system with sand drains, degree of consolidation is different at each location along the length. The rate of settlement with time in the presence of sand drains is greater than without these. This is due to the fact that the consolidation at and near the center of the drain is very fast and a greater degree of consolidation is reached at a small time. Figure 8(b) shows the effect of radius of sand drain with a constant normalized time factor 20 and all other parameters kept constant. It is observed that as the radius of sand drain increases, settlement also increases at every point. Larger radius of sand drain induces greater degree of consolidation, and hence greater settlement.

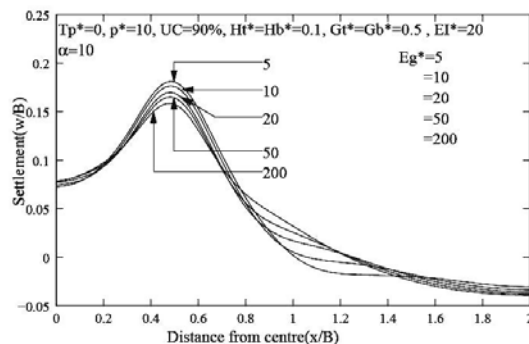


Fig. 3(a). Settlement profiles for various values of modulus of geosynthetic membrane

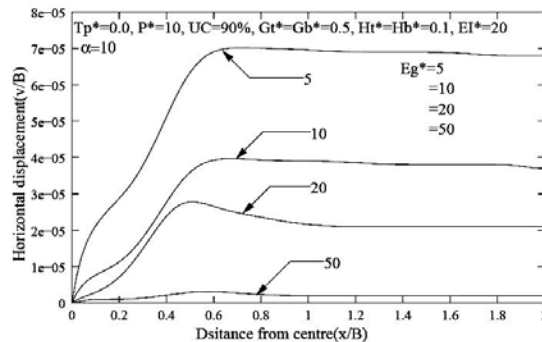


Fig. 3(b). Horizontal displacement profiles for tension various values of tension modulus

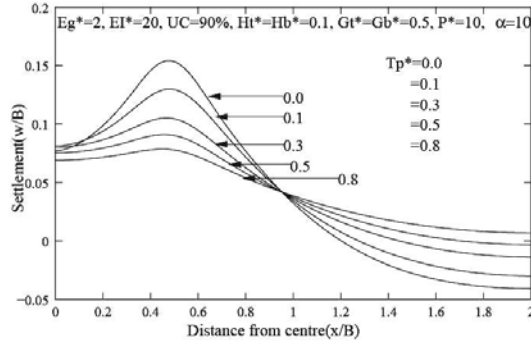


Fig. 4. Settlement profiles for various values of pretension

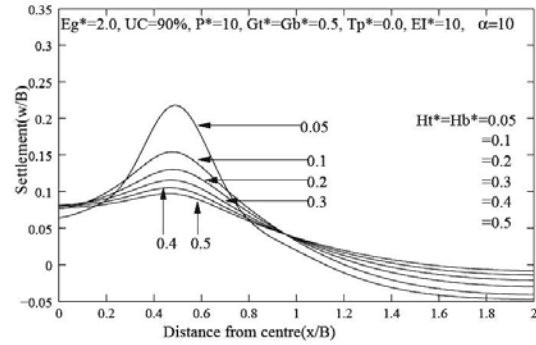


Fig. 5. Settlement profiles for various values of thickness of granular fill

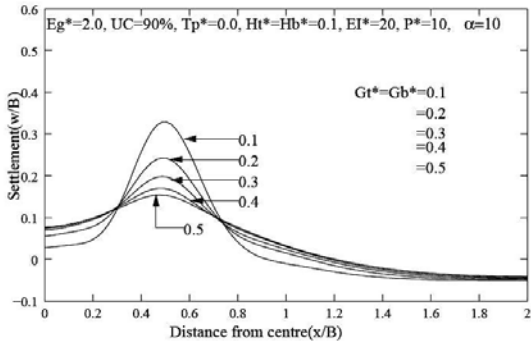


Fig. 6. Settlement profiles for various values of shear modulus of granular fill

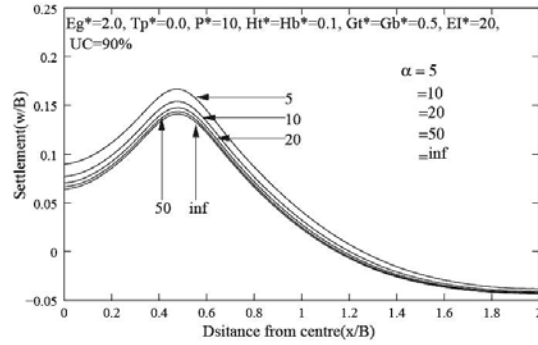


Fig. 7. Settlement profiles for various values of spring constant ratio

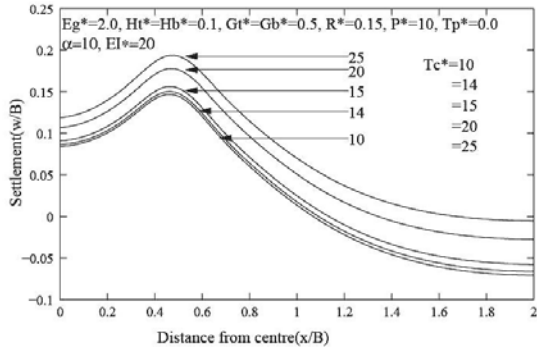


Fig. 8(a). Settlement profiles for various values of normalized time

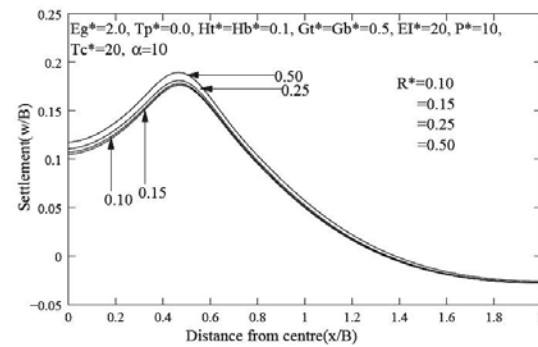


Fig. 8(b). Settlement profiles for various values of radius of sand drain

#### 4 Conclusions

The paper presents the settlement analysis of a beam resting on geosynthetic-reinforced granular fill-soft soil system. Each sub-system of the reinforced fill soft soil system is idealized by the mechanical foundation model elements, such as rough elastic membrane, Pasternak shear layer, Winkler springs and dashpots. The differential equations governing the settlement response of the beam resting on two layered reinforced foundation soil have been formulated by incorporating the deformation compatibility conditions. The numerical solutions are obtained using finite element method and results are presented in non-dimensional form. It is observed that the variation in the tension modulus in the geosynthetic membrane does not appreciably affect the settlement response of the granular fill-soft soil system under the range of parameters studied. Pretension in the geosynthetic membrane appreciably affects the settlement response of the system. Settlement gets reduced below the point of application of load; however, no appreciable change is noticed at the centre of the system. A magnitude greater than 0.8 for

the pretension force results in no-heaving condition at the edges of the beam. Pretension force is found to be effective in reduction of differential settlement. Increasing the thickness of granular fill resulted in reduction of the settlement of the system quite effectively. Increment in the shear modulus of the granular fill resulted in the reduction of settlement below the point of application of load, while the settlement increased at the centre and at the edge of the beam. It is observed that higher magnitudes of shear modulus are preferable to reduce differential settlement. Beyond a magnitude of 0.4, the shear modulus of granular fill has no effect on the settlement characteristics of the system. The spring constant ratio of the granular fill is studied to bring out the effect of relative compressibility of the granular fill and the soft soil on the settlement behavior. It is observed that when the stiffness of the granular fill exceeded 50 times that of the soft soil, the effect of the compressibility of the granular fill is negligible. It is observed that in a system without sand drains, the consolidation is uniform along the length of the system; however, introduction of sand drains in the system resulted in accelerated and different degrees of consolidation at each location of the system. It is noticed that the radius of the sand drain has an appreciable effect on the achieved degree of consolidation; a larger radius induced a higher degree of consolidation and hence resulted in a higher degree of settlement. In this study, it is observed that the horizontal displacement of the geosynthetic membrane is negligible as compared to the vertical settlement.

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