

INTERNATIONAL MATHEMATICS OLYMPIAD

The actual test paper has 50 questions. Time allowed : 60 minutes. There are 3 sections, 20 questions in section I, 20 in section II and 10 in section III.

Section I : Logical Reasoning, **Section II** : Mathematical Reasoning & **Section III** : Everyday Mathematics

SYLLABUS

Relation and function, Inverse trigonometric function, Matrices, Determinants, Continuity and differentiability, Application of derivatives, Integrals (Definite and indefinite), Application of integrals, Differential equations, Vector algebra, Three Dimensional geometry, Linear programming, Probability, Differentiation.

LOGICAL REASONING

1. The “Golden Rectangle” of the ancient Greeks was considered to have the most pleasing proportion of any rectangle. The ratio of width (w) to height (h) of the rectangle is expressed in the following proportion and is shown in the drawing below.

$$\frac{w}{h} = \frac{2}{\sqrt{5} - 1}$$



Jason is planning to paint a rectangular mural using the proportions of the “Golden Rectangle.” If the mural is 15 meters wide, how high should it be?

- (A) 1.6 meters (B) 9.3 meters (C) 16.5 meters (D) 24.2 meters (E) None of these.

2. The given table shows the boiling points in degrees Celsius for some different elements.

BOILING POINTS OF SOME ELEMENTS

Element	Boiling Point (in °C)
Chlorine	– 34.6
Helium	– 269.0
Hydrogen	– 252.9
Nitrogen	– 195.8
Oxygen	– 183.0

Which of the following elements have boiling points that are lower than –190°C ?

- (A) chlorine and oxygen (B) oxygen and nitrogen
(C) chlorine, helium, and hydrogen
(D) helium, hydrogen, and nitrogen (E) None of these.

3. A certain radioactive element decays over time according to the equation $y = A \left(\frac{1}{2}\right)^{\frac{t}{300}}$, where A = the number of grams present initially and t = time in years. If 1000 grams were present initially, how many grams will remain after 900 years?

- (A) 500 grams (B) 250 grams (C) 125 grams (D) 62.5 grams (E) None of these.

4. Which is the first incorrect step in simplifying $\log_4 \frac{4}{64}$?

Step 1: $\log_4 \frac{4}{64} = \log_4 4 - \log_4 64$

Step 2 : $= 1 - 16$

Step 3 : $= - 15$

- (A) Step 1 (B) Step 2 (C) Step 3
(D) Each step is correct (E) None of these.

5. Which of the following sentence is true about the graphs of $y = 3(x - 5)^2 + 1$ and $y = 3(x + 5)^2 + 1$?

- (A) Their vertices are maximum (B) The graphs have the same shape with different vertices
(C) The graphs have different shapes with different vertices
(D) One graph has a vertex that is a maximum, while the other graph has a vertex that is a minimum
(E) None of these.

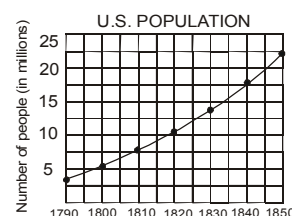
6. If \vec{a} , \vec{b} , \vec{c} are vectors such that $\vec{c} = \vec{a} + \vec{b}$ and $\vec{a} \cdot \vec{b} = 0$ then

- (A) $a^2 + b^2 + c^2 = 0$ (B) $a^2 - b^2 = c^2$
(C) $a^2 + b^2 = c^2$ (D) $\vec{c} = \vec{a} \times \vec{b}$ (E) None of these.

7. In 1790, the United States had a population of approximately 4,000,000 people, as shown on the graph below.

According to the graph, in what year had the population grown to approximately twice that number?

- (A) 1797 (B) 1808 (C) 1813
(D) 1822 (E) None of these.



MATHEMATICAL REASONING

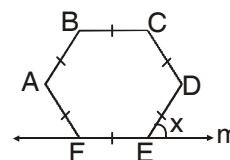
8. Which expression represents $f(g(x))$ if $f(x) = x^2 - 1$ and $g(x) = x + 3$?

- (A) $x^3 + 3x^2 - x - 3$ (B) $x^2 + 6x + 8$ (C) $x^2 + x + 2$ (D) $x^2 + 8$ (E) None of these.

9. From a deck of card two are drawn. The probability that both are of same suit is

- (A) $\frac{1}{2}$ (B) $\frac{1}{13}$ (C) $\frac{4}{17}$ (D) $\frac{2}{17}$ (E) None of these.

10. Figure **ABCDEF** below is a regular hexagon with line m passing through side FE . What is the measure $\angle x$?
- (A) 75° (B) 60° (C) 51°
 (D) 45° (E) None of these.



11. On a recent test, Jyoti wrote the equation $\frac{x^2 - 16}{x - 4} = x + 4$. Which of the following statements is correct about the equation she wrote?
- (A) The equation is always true (B) The equation is always true, except when $x = 4$
 (C) The equation is never true (D) The equation is sometimes true when $x = 4$
 (E) None of these.

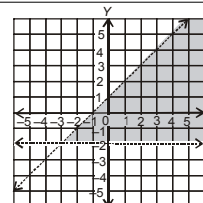
12. If x is a real number, which best describes the values of x for which the inequality $\sqrt{x} > 0$ is true?
- (A) all $x > 0$ (B) all $x \geq 0$
 (C) all values of x (D) no values of x (E) None of these.

13. Given the equation $y = x^n$ where $x > 0$ and $n < 0$, which statement is valid for real values of y ?
- (A) $y > 0$ (B) $y = 0$ (C) $y < 0$ (D) $y \neq 0$ (E) None of these.

14. If the equation $y = 2^x$ is graphed, which of the following values of x would produce a point closest to the x -axis?
- (A) $1/4$ (B) $3/4$ (C) $5/3$ (D) $8/3$ (E) None of these.

15. The graph of $\left(\frac{x}{2}\right)^2 - \left(\frac{y}{3}\right)^2 = 1$ is a hyperbola. Which set of equations represents the asymptotes of the hyperbola's graph?
- (A) $y = \frac{3}{2}x$, $y = -\frac{3}{2}x$ (B) $y = \frac{3}{2}x$, $y = -\frac{2}{3}x$
 (C) $y = \frac{1}{2}x$, $y = -\frac{1}{2}x$ (D) $y = \frac{1}{3}x$, $y = -\frac{1}{3}x$ (E) None of these

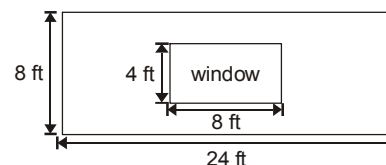
16. What system of inequalities best represents the graph shown below?
- (A) $y > -2$ and $y > x + 1$ (B) $y > -2$ and $y < x + 1$
 (C) $y < -2$ and $y > x + 1$ (D) $y < -2$ and $y < x + 1$
 (E) None of these.



17. If $\int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx = \frac{k}{4}$ then value of k equals
- (A) $\pi/12$ (B) $\pi/3$ (C) $\pi/2$ (D) $\pi/7$ (E) None of these

EVERYDAY MATHEMATICS

18. Mrs. Ballard decided to apply wallpaper on one wall of her living room. A diagram of the rectangular wall with its window is shown below. A roll of wallpaper covers approximately 30 square feet. What is the minimum number of rolls she will have to buy in order to cover the entire wall excluding the window?



- (A) 2 rolls (B) 5 rolls (C) 6 rolls (D) 7 rolls (E) None of these.
19. A box contains 7 large red marbles, 5 large yellow marbles, 3 small red marbles, and 5 small yellow marbles. If a marble is drawn at random, what is the probability that it is yellow, given that it is one of the large marbles?
- (A) $5/12$ (B) $7/20$ (C) $5/8$ (D) $1/5$ (E) None of these.
20. A restaurant manager bought 20 packages of bagels. Some packages contained 6 bagels each, and the rest contained 12 bagels each. There were 168 bagels in all. How many packages of 12 bagels did the manager buy?
- (A) 6 (B) 8 (C) 9 (D) 12 (E) None of these.

ANSWER KEY

1. (B) 2. (D) 3. (C) 4. (B) 5. (B) 6. (C) 7. (C) 8. (B) 9. (B) 10. (B)
 11. (B) 12. (A) 13. (A) 14. (A) 15. (A) 16. (B) 17. (B) 18. (C) 19. (A) 20. (B)